Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MFP2

Unit Further Pure 2

Wednesday 18 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express -9i in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

[2 marks]

(b) Solve the equation $z^4 + 9\text{i} = 0$, giving your answers in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

2 (a) Sketch, on the Argand diagram below, the locus L of points satisfying

$$\arg(z - 2\mathrm{i}) = \frac{2\pi}{3}$$

[3 marks]

(b) (i) A circle C, of radius 3, has its centre lying on L and touches the line $\operatorname{Im}(z) = 2$. Sketch C on the Argand diagram used in part **(a)**.

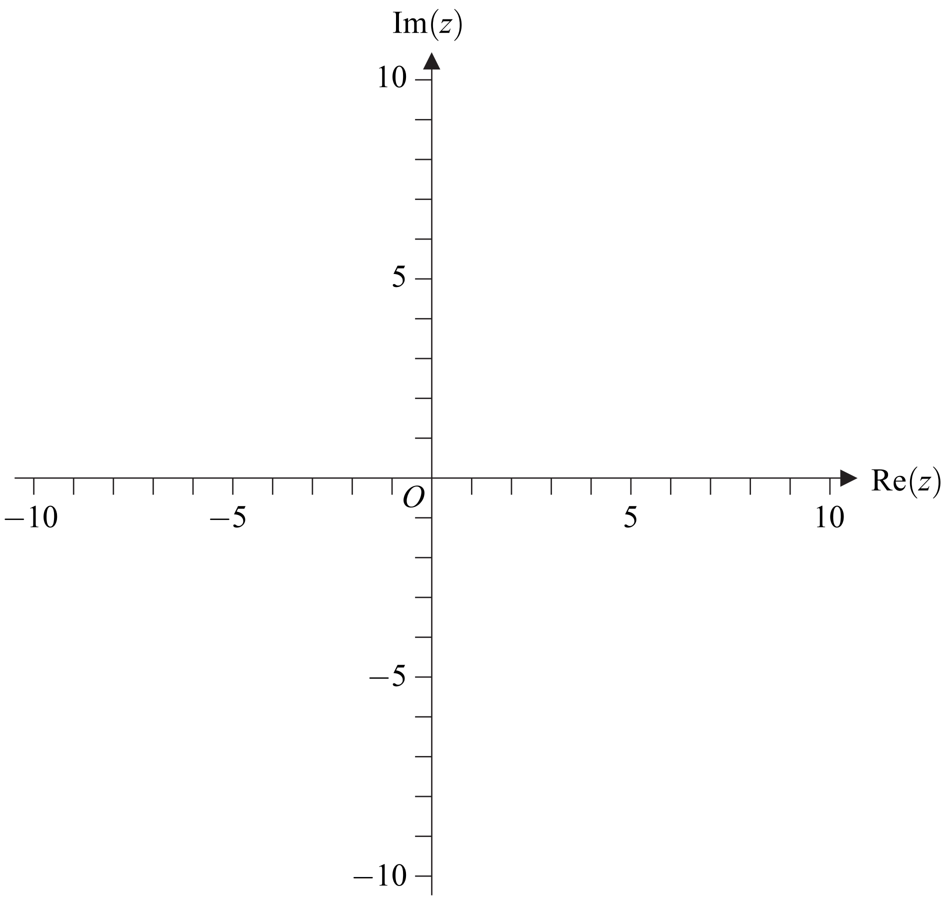
[2 marks]

(ii) Find the centre of C, giving your answer in the form $a + b\mathrm{i}$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2



QUESTION PART REFERENCE

Answer space for question 2

3 (a) Express $(k+1)^2 + 5(k+1) + 8$ in the form $k^2 + ak + b$, where a and b are constants.

[1 mark]

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} r(r+1)\left(\frac{1}{2}\right)^{r-1} = 16 - \left(n^2 + 5n + 8\right)\left(\frac{1}{2}\right)^{n-1}$$

[6 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

4 The roots of the equation

$$z^3 + 2z^2 + 3z - 4 = 0$$

are α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.

[2 marks]

(ii) Hence show that $\alpha^2 + \beta^2 + \gamma^2 = -2$.

[2 marks]

(b) Find the value of:

(i) $(\alpha + \beta)(\beta + \gamma) + (\beta + \gamma)(\gamma + \alpha) + (\gamma + \alpha)(\alpha + \beta)$;

[3 marks]

(ii) $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$.

[4 marks]

(c) Find a cubic equation whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

5 (a) Using the definition $\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta})$, prove the identity

$$4\sinh^3\theta + 3\sinh\theta = \sinh 3\theta$$

[3 marks]

(b) Given that $x = \sinh\theta$ and $16x^3 + 12x - 3 = 0$, find the value of θ in terms of a natural logarithm.

[4 marks]

(c) Hence find the real root of the equation $16x^3 + 12x - 3 = 0$, giving your answer in the form $2^p - 2^q$, where p and q are rational numbers.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

6 (a) (i) Use De Moivre's Theorem to show that if $z = \cos\theta + \mathrm{i}\sin\theta$, then

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

[3 marks]

(ii) Write down a similar expression for $z^n + \dfrac{1}{z^n}$.

[1 mark]

(b) (i) Expand $\left(z - \dfrac{1}{z}\right)^2 \left(z + \dfrac{1}{z}\right)^2$ in terms of z.

[1 mark]

(ii) Hence show that

$$8\sin^2\theta\cos^2\theta = A + B\cos 4\theta$$

where A and B are integers.

[2 marks]

(c) Hence, by means of the substitution $x = 2\sin\theta$, find the exact value of

$$\int_1^2 x^2\sqrt{4 - x^2}\,\mathrm{d}x$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

7 (a) Given that $y = \tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ and $x \neq 1$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{1+x^2}$.

[4 marks]

(b) Hence, given that $x < 1$, show that $\tan^{-1}\left(\dfrac{1+x}{1-x}\right) - \tan^{-1} x = \dfrac{\pi}{4}$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

Turn over ▶

8 A curve has equation $y = 2\sqrt{x-1}$, where $x > 1$. The length of the arc of the curve between the points on the curve where $x = 2$ and $x = 9$ is denoted by s.

(a) Show that $s = \int_2^9 \sqrt{\frac{x}{x-1}}\, dx$.

[3 marks]

(b) (i) Show that $\cosh^{-1} 3 = 2\ln(1+\sqrt{2})$.

[2 marks]

(ii) Use the substitution $x = \cosh^2\theta$ to show that

$$s = m\sqrt{2} + \ln(1+\sqrt{2})$$

where m is an integer.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED